Benha University Faculty of Engineering – Shoubra Department of Industrial Engineering Duration : 2 hours **Group K**		Final Exam Course: Mathematics 2 Code: EMP 102 Date: May 12 , 2018

The exam consists of one page   No. of questions: 4   Answer **All** questions   Total Mark: 40

Question 1

(a)Find **y`** from the following: **4**

(i) $y = \cosh x \,.\, \sin^{-1} x + \tanh x$ (ii) $y = \sinh^{-2} x + \tan^{-1} x \,.\, \cosh^{-1} x$

(b)Find the integrals: **6**

(i)$\int (x^4 - 4^x)\, dx$ (ii)$\int (\frac{1}{x-2} + \frac{1}{1+x^2})\, dx$ (iii)$\int \frac{x-1}{x^2-3x-4} dx$

(iv)$\int \cosh^{-1} x\ dx$ (v) $\int (\sinh x + \sin^2 x)\, dx$ (vi)$\int (\cosh x + \cos^2 2x)\, dx$

Question 2

(a)Find the area of the region between the curve $y = x^3 - x$, x-axis, x in [0, 3]. **2**

(b)Find the arc length of the curve : $y = e^x$, x in [0, 2]. **2**

(c)If the region between the curve $y = x + \sqrt{x}$, x-axis, x in [2, 3] is rotated about **4**
(i) x-axis (ii)y-axis. Find the volume of the generated solids V_x , V_y.

(d)If the arc of the curve $y = \ln x$, x in [2, 3] is rotated about y-axis. **2**
Find the surface area S_y.

Question 3

(a)State the definition of : (i)Line (ii)Parabola (iii) Ellipse. **3**

(b)Separate the lines : $x^2 - 2xy - 3y^2 = 0$ and then find the angle between them. **2**

(c)Find the points of intersection of circles : **2**
$x^2 + y^2 - 3x - y + 2 = 0,\ x^2 + y^2 - 2x - 2y + 2 = 0.$

(d)Write the equation of parabola whose focus (2, 1) and directrix $y + 3 = 0$. **2**

Question 4

(a)Find the vertex , focus and sketch the parabola : $x^2 - 2x - 12y + 13 = 0$. **3**

(b)Find center, vertices and sketch the ellipse : $x^2 + 4y^2 - 2x + 8y + 4 = 0$. **3**

(c)Find center, vertices and sketch the hyperbola : $x^2 - y^2 - 4x + 13 = 0$. **3**

(d)Determine the type of the curve : $x^2 - 4xy - y^2 + 4x - 8y + 4 = 0$. **2**

Good Luck   *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)(i) $y` = \sinh x \,.\, \sin^{-1} x + \cosh x \,.\, \frac{1}{\sqrt{1-x^2}} + \operatorname{sech}^2 x$

(ii) $y` = -2(\sinh x)^{-3} . \cosh x + \frac{1}{1+x^2} . \cosh^{-1} x + \tan^{-1} x \,.\, \frac{1}{\sqrt{x^2-1}}$

--*4- Marks*

(b)(i) $I = \frac{x^5}{5} - \frac{4^x}{\ln 4} + c$

(ii) $I = \ln(x-2) + \tan^{-1} x + c$

(iii) $\int \frac{x-1}{x^2-3x-4} dx = \int (\frac{\frac{3}{5}}{x-4} + \frac{\frac{2}{5}}{x+1}) dx = \frac{3}{5} \ln(x-4) + \frac{2}{5} \ln(x+1) + c$

(iv) $I = x \cosh^{-1} x - \int \frac{x}{\sqrt{x^2-1}} dx = x \cosh^{-1} x - \sqrt{x^2-1} + c$

(v) $I = \int (\sinh x + \frac{1}{2}(1 - \cos 2x))\, dx = \cosh x + \frac{1}{2}(x - \frac{1}{2} \sin 2x) + c$

(vi) $I = \int (\cosh x + \frac{1}{2}(1 + \cos 4x))\, dx = \sinh x + \frac{1}{2}(x + \frac{1}{4} \sin 4x) + c$

--*6- Marks*

Answer of Question 2

(a)From $x^3 - x = 0$, we get x = 0, 1 , – 1. We see that 1 inside the interval and 0, – 1 outside the interval [0, 3].

Then $A = \int_0^1 (x^3 - x)dx + \int_1^3 (x^3 - x)dx = |-\frac{1}{4}| + 16 = \frac{65}{4}$

--*2- Marks*

(b)The arc length $L = \int_0^2 \sqrt{1 + e^{2x}}\, dx = 6.79$

--*2- Marks*

(c)(i) $V_x = \pi \int_2^3 (x + \sqrt{x})^2\, dx = 16.78\,\pi$

(ii) $V_y = 2\pi \int_2^3 x \left(x + \sqrt{x}\right) dx = 20.61\,\pi$

--*4- Marks*

(d) $S_y = 2\pi \int_2^3 x \sqrt{1 + \frac{1}{x^2}}\, dx = 5.39\,\pi$

--*2- Marks*

Answer of Question 3

(a)Definitions: Line , Parabola , Ellipse.

--*3- Marks*

(b) $x^2 — 2xy - 3y^2 = (x + y)(x - 3y) = 0.$

The two lines are : x + y = 0 , x – 3y = 0 and $\tan\theta = \frac{-1-\frac{1}{3}}{1-\frac{1}{3}} = 2$.

--*2- Marks*

(c)The radical axis is : – x + y = 0 Or y = x. Substitute in the first circle, we get the equation $x^2 - 2x + 1 = 0$. Then x = 1 = y and the point of intersection is (1, 1).

--*2- Marks*

(d)The equation of parabola is : $\sqrt{(x-2)^2 + y - 1^2} = \frac{y+3}{1}$.

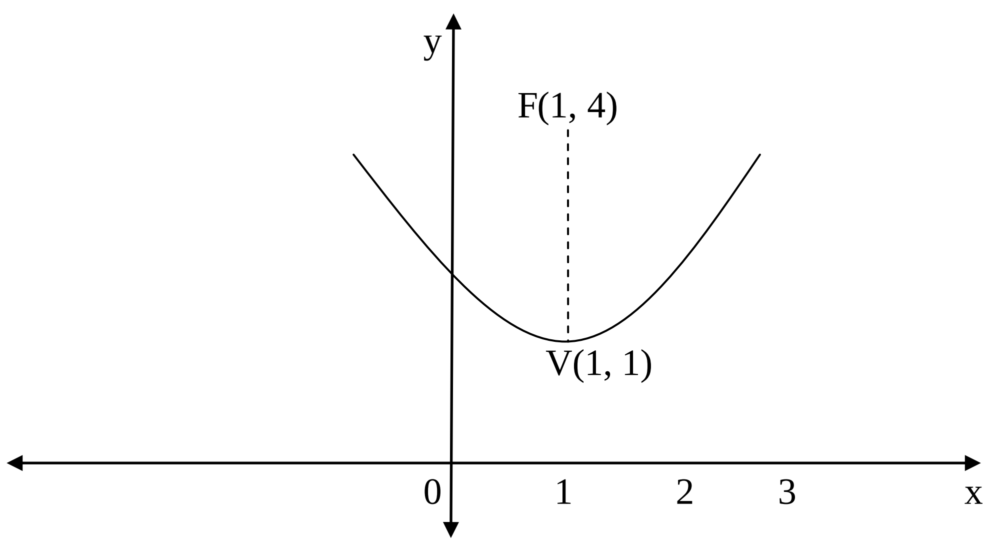
Or $x^2 - 4x - 8y - 4 = 0$

--*2- Marks*

Answer of Question 4

(a)From : $x^2 - 2x - 12y + 13 = 0$, we get $(x-1)^2 = 12(y-1)$

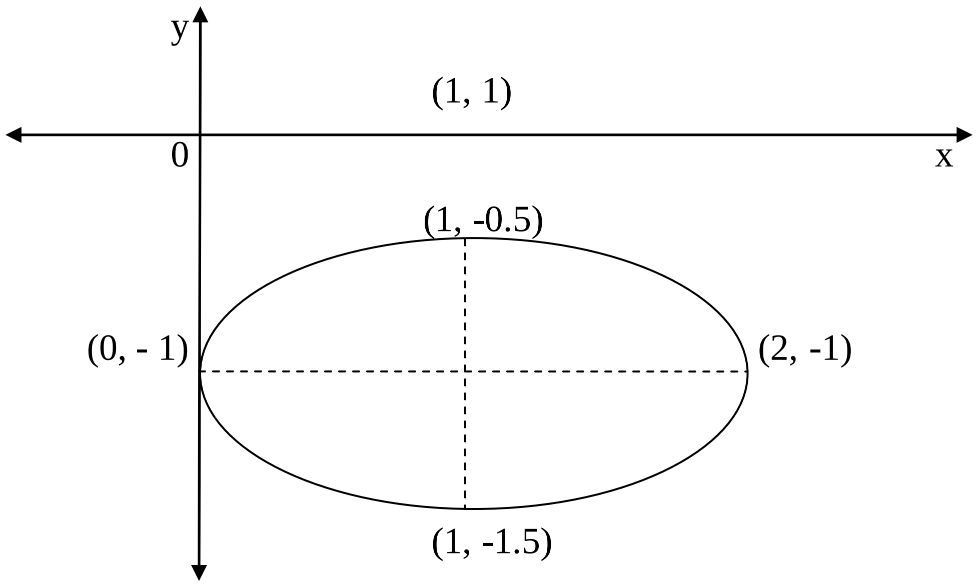
The vertex is (1, 1) , a = 3, vertical.



--*3- Marks*

(b)From : $x^2 + 4y^2 - 2x + 8y + 4 = 0$, we get $\frac{(x-1)^2}{1} + \frac{(y+1)^2}{1/4} = 1$

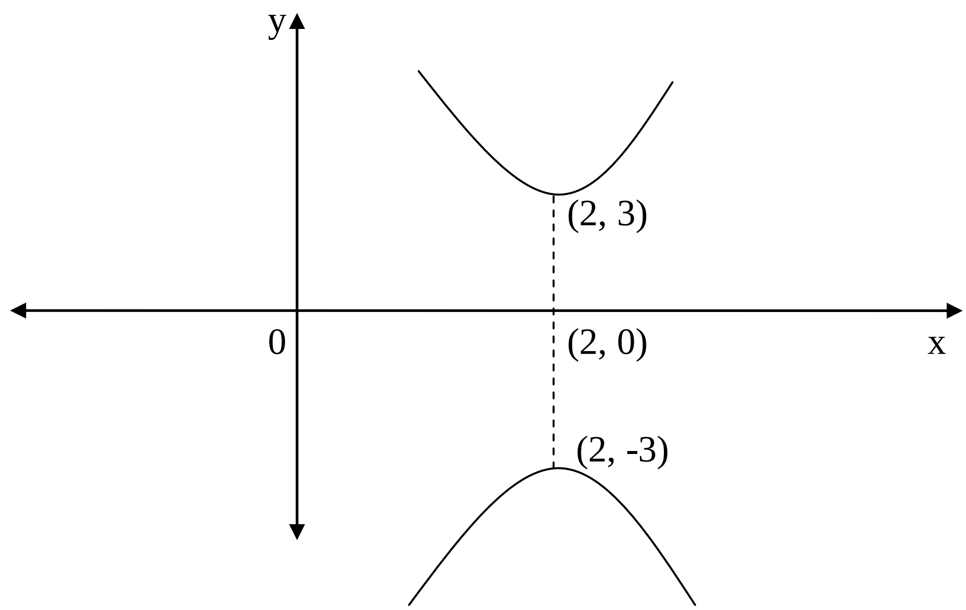
The center is (1, -1) , a = 1, b = 1/2, horizontal.



--*3- Marks*

(c)From : $x^2 - y^2 - 4x + 13 = 0$, we get $\frac{y^2}{9} - \frac{(x-2)^2}{9} = 1$

The center is (2, 0) , b = 3, vertical.



--*3- Marks*

(d) $\Delta = \begin{vmatrix} 1 & -2 & 2 \\ -2 & -1 & -4 \\ 2 & -4 & 4 \end{vmatrix} = 0$. It is pair of lines.

--*2- Marks*

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